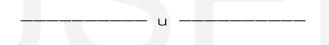
# Expressing Filter Press Specific Resistance as a function of Cake Yield using LMT Dimensional Analytical Approach

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Abstract - An equation enabling prediction of specific resistance to filter pressing of sludge has been derived using Buckingham- $\pi$  method of LMT Dimensional Analysis. The model equation developed (Equation 27) shows that the specific resistance to cake filtration for filter press is directly proportional to the filter area of the pressure vessel, applied pressure and the compressibility of the sludge while being inversely proportional to filter cake yield, viscosity of filtrate and the initial solids content of the sludge. This is in agreement with scientific reasoning and experimental observation. The new equation enables performance of a pressure filter (Filter Press) to be predicted from a simple laboratory determination of both specific resistance and cake yields. Curves derived from model show how specific resistance to pressure filtration of cake depends on a number of parameters, for instance, figures 4,5,6,7 and 8 respectively illustrates the effects of pressures, conditioner dosages, initial solids contents, cake yield and compressibility on the specific resistance. For instance, increasing ferric chloride dosage from 22.61% to 28.08% reduced specific resistance for 6628.18g/Cm<sup>2</sup>. Increasing ferric chloride dosage from the graph to attain acceptable filtrate quality was 22% at an operating pressure of 6628.18g/Cm<sup>2</sup>. Increasing ferric chloride dosage beyond this optimum value did not further reduce specific resistance to filtration anymore. Moreover, experimental verification of the equation and the derived curves has been described. Although, the scope of the experimental verification was somewhat limited, it was sufficient enough to demonstrate the use of the equation and curves derived in this study. The method can be applied in predicting full-scale performance for any sludge from laboratory experiments and the procedure has been fully described and comparatively analyzed.

Keywords- Specific resistance, Cake yield, Sludge, Expressing, Filter press, Dimensional Analysis. Filtration,



#### 1. Introduction

The management of sludge from both domestic and industrial process operations occasioned by modern advancement in technology is highly complex and cost effective, hence, if poorly accomplished, may jeopardize the environmental and sanitary advantages expected from the treatment. The importance attached to sludge management was widely acknowledged by Agenda 21, which included in the theme of environmentally wholesome management of solid wastes; sludge issues and as well defined the following modalities towards its administration; reduction in production, maximum increase in reuse and recycling and the adoption of environmentally wholesome treatment and disposal [1].

Moreover, due to the low indices of wastewater and other industrial processes resulting from high population increase and industrialization especially in developing countries, there is need to explore and adopt more scientific options to meet the demand. As a consequence, the amount of sludge produced is expected to increase posing serious environmental and health concerns to the developing nations [2],[3].

[4] had stated that Wastewater treatment processes such as activated sludge treatment systems produce surplus sludge which has to be disposed of. The surplus sludge contains valuable nutrients and organic matter that can be used to improve soil quality and as a fertilizer for agricultural crops. Sludge treatment processes generally have two main purposes: (i) thickening and dewatering whereby the sludge volume and hence the costs of subsequent handling, transportation, and disposal are reduced [5].

A recent study conducted by [6] and [7] revealed that despite the fact that the volume of sludge tends to be less than 1% of the total plant influent, sludge handling costs ranges between 21-50% of total plant operation and maintenance costs,. Dewatering of sewage sludge is not only found in removal of the excess moisture but to render the sludge odourless and nonputrescible, [8]. [9] and [10] maintained that dewatering of sewage sludge prior to drying or disposal is an important step because the lower the water content of the sludge, the lesser the transport costs. Proper waste management system should be

established and enhanced in view of menace imposed in our community due to improper handling and disposal of wastes to our environment.

Dimensional Analyses is a conceptual tool often applied in physics, chemistry and engineering to understand physical solution involving a mixture of different kinds of physical quantities. Dimensional formulae provide a useful catalogue system for physical quantities according to [11]. The principle of dimensional homogeneity states that in a physical equation consisting of an algebraic sum of two or more terms, the exponent of the dimension of Length, Mass and Time in any term of the equation must be the same as that in any other term. The system of fundamental units commonly used in Newtonian mechanics is the LMT System.

In this study, Buckingham  $\pi$ -method was used in developing the new model. Assigning any arbitrary value to the exponent of the variables of interest and expressing the variable as a product of others became expedient hence,

$$Y = P^a A^b \mu^c C^d V^e R^f \theta^g S^h \dots (1)$$

# 1.1. A Panoramic review of Previous Works on Specific Resistance Models

Researchers over the years have come up with different equations to express the resistance of processing fresh sludge into sludge cake. Carman in a study in 1934 postulated a filtration equation whereby an assumption was made that the specific resistance is constant throughout the sludge cake thickness and that the cake is rigid. [12] and [13] disagreed with Carman's equation in that the specific resistance parameter should be designated as an average value.

The essence of the research was aimed at formulating a new cake yield equation with both compressibility and filter yield attributes as measures of filterability. The incorporation of the compressibility coefficient 'S' is against the traditional filtration equations already suggested by [14],[15],[16] and [17] where the sludge compressibility effects on filterability was obviously unaccounted for. It has been discovered in literature that traditional equations were embedded uncertainties in the areas of formulating them. [18] stressed that since the literature is replete of dewatering operations which have unsatisfactory performance predictions and formulations and considering the controversies among prominent researcher to the present knowledge of filtration equations, it is justified that an acceptable equation which characterize the filtration process has to be derived. The equation to be derived must contain the compressibility coefficient 'S' as an attribute. The incorporation of 'S' will make such equation acceptable to the previous researchers.

Carman derived his equation based on non-compressible sludge cakes. In his equation, which was a modification of Darcy's equation, he stressed that the specific resistance is constant throughout the filtration process. Hence, [14] proposed the equation

$$\frac{dV}{d\theta} = \frac{P.A^2}{\mu (RCV + R_m A)} \dots (2)$$

Where:

A = filter area, m<sup>2</sup>

V= volume of filtrate, m<sup>3</sup>

 $P = \text{pressure drop,Kg/m}^2$ 

C = concentration of solids in the feed, Kg/m<sup>3</sup>

R = specific cake resistance, kg/m

μ = liquid viscosity, poise

 $\theta$  = filtration time, s

R<sub>m</sub> = septum resistance, kg/m

Integrating the above and neglecting septum resistance, He obtained an expression for specific resistance, R given as:

$$R = (\frac{2bPA^2}{\mu C})$$
 ......(3)

Where C is the initial solids content (g/Cm³) and b is the slope of the plot of t/V against V.

[19] developed a model to examine the effects of process parameters affecting sludge dewatering using Diaphragm Filter Press. It was discovered that Diaphragm Filter Press filtration process occurs in two distinct steps: the filtration time and the squeezing time. While the filtration phase follows the known conventional pattern at the terminal pressures ranging from 56 to 98Psi, squeezing occurs at terminal pressure of 215Psi. They listed initial solid contents, conditioning, operating pressures and times among process parameters affecting filterability.

The introduction of compressibility coefficient 'S' and filter cake yield as attributes of the new equation cannot be over emphasized. Firstly it eases the rigorous mathematical manipulation of maximum specific resistance as a means of

determining the filterability of sludge cakes as formulated by the above named researchers. Similarly, in view of the foregoing, [20] in the formulation of a valid equation, stated that compressibility attribute of the sludge cake in question should be properly accounted for since the Poiseuille and Darcy laws representing the basis of formulation are only applicable to compressible sludge cakes (rigid materials). It is in the light of this foregoing postulations, that this new equations has judiciously accounted for compressibility coefficient 'S' bringing the total variables to nine in the formulation of the new equations for compressible sludge cakes.

#### Materials and Method

## 2.1 Preparation of Sludge Slurry

A photograph of the Filter Press apparatus is seen in figure 1 while the schematic diagram is as shown in figure 2.

Brewery sludge slurry was prepared by mixing given quantities of the desired sludge in 1 liter of distilled water and agitating it for few minutes. 60 ml was drawn from the slurry using a pre-weighed cylinder. The weight of the cylinder and slurry was measured and recorded as  $w_1$ . The slurry was then oven dried at 103°C for 24hours. The new weight was thereafter noted and recorded as w2. The concentration of the slurry was then calculated as fully described. The filtration process was started by connecting the compressed air line to the top of the sample holder using an easy push through arrangement. Thus, the suspension was forced to flow through the filter cell producing a filter cake at the surface of the filter paper. As the filter cake deposited, the flow rate kept declining. The filtrate was collected in a graduated cylinder placed in a tilted position so that the filtrate traveled along the walls of the cylinder without causing a splash, enabling accurate determinations of equal increments of the filtrate volume. After about 60 ml of the suspension had filtered through and had formed a cake on the surface of the filter paper,

After about 60 ml of the suspension had filtered through and had formed a cake on the surface of the filter paper, time increments required to produce successive equal increments of filtrate volume were observed. The data were plotted in the appropriate manner and both the cake yield and the specific resistance of the filter cake were then calculated using equation 27.

Constant pressure filtration experiments were carried out respectively  $0\,n$   $0.0128\,g/C\,m^3$ ,  $0.0194\,g/C\,m^3$ .  $0.0220\,g/C\,m^3$ ,  $0.02465\,g/C\,m^3$  and  $0.0393\,g/C\,m^3$  sludge samples using different doses of Ferric Chloride suspensions. The filtration pressure ranges investigated was between  $2039.43g/Cm^2-6628.155g/Cm^2$ . Filtration was allowed to proceed and stopped once deliquoring, which was determined when

the t/V against V plot experienced a sudden change in accordance to traditional filtration behavior was deemed to be beginning. On the conclusion of filtration the formed cake in the filter cell was removed and put in a preweighed beaker. It was then dried over a period of twenty four hours and reweighed. The raw data from the laboratory, pilot filter runs made to evaluate the effect of pressure drop, initial solids contents, conditioner dosages, specific resistance and compressibility on cake yields have been analyzed.



Figure 1: The Filter Press Assembly

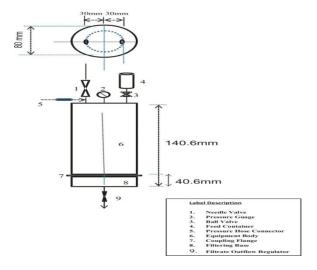


Fig. 2: Schematic Diagram of a Filter Press Apparatus

### 2.2. Developing the New Filtration Equation

In order to derive the new equation, the Buckingham's  $\pi$ -method of dimensional analysis was employed. The sludge cake yield (Y) is a function of volume of the sludge (V), filter paper area (A), time of filtration ( $\theta$ ), mass of solids per unit volume of filtrate (C), net filtration pressure (P), viscosity of filtrate ( $\mu$ ), the average specific resistance of filter cake (R) and Sludge Compressibility (S). This is mathematically expressed as Equation (4). Table 1 is a summary of the relevant variables and their dimensions as applied in this derivation.

$$Y = f(P, A, C, V, \mu, \theta, R, S) \qquad .... \qquad ... \qquad .$$

From the Buckingham's  $\pi$ -method theories, the total number of variables (n) is nine while the number of fundamental dimensions (m) is three,

Hence, the number of  $\pi$ - terms is n - m, 9 - 3 = 6.

Therefore, number of  $\pi$ -terms in the equation can be written as:

$$f(\pi_{1}, \pi_{2}, \pi_{3}, \pi_{4}, \pi_{5}, \pi_{6}) = 0 \qquad (6)$$

$$\pi_{1} = P^{a} A^{b} \mu^{c} Y \qquad (7)$$

$$\pi_{2} = P^{a} A^{b} \mu^{c} R \qquad (8)$$

$$\pi_{3} = P^{a} A^{b} \mu^{c} C \qquad (9)$$

$$\pi_{4} = P^{a} A^{b} \mu^{c} \theta \qquad (10)$$

$$\pi_{5} = P^{a} A^{b} \mu^{c} V \qquad (11)$$

$$\pi_{6} = P^{a} A^{b} \mu^{c} S \qquad (12)$$

Where  $\pi_1$  to  $\pi_6$  are dimensionless terms while a, b, and c are exponents to be determined by dimensional Analysis.

Table 1: Summary of LMT Dimensional formula

Physical Variables	Symbols	Dimensions
Yield	Y	ML-2T-1
Volume	V	<b>L</b> 3
Filtration Area	А	$L^2$
Time for Filtration	θ	Т
Mass of cake dry solids per unit	С	ML <sup>-3</sup>
Net filtration Pressure	Р	ML <sup>-1</sup> T <sup>-2</sup>
Viscosity of filtrate	μ	ML <sup>-1</sup> T <sup>-1</sup>
Average specific resistance of	R	LM <sup>-1</sup>
Compressibility coefficient	S	м <sup>-1</sup> LТ <sup>2</sup>

Considering  $\pi$ 1– Term

By replacing the right hand side of equation (7) with the corresponding dimensions of the variables and the dimensionless term on the left hand side with  $M^0$   $L^0$   $T^0$ , equation obtained is given as:

$$M^{0} L^{0} T^{0} = (ML^{-1}T^{-2})^{a} (L^{2})^{b} (ML^{-1}T^{-1})^{c} (ML^{-2}T \dots (13)$$

Where a, b, c, are unknowns to be determined using dimensional homogeneity between variables.

Equating the exponents of M, L and T on the left hand side to the corresponding exponents on the right hand side, we get,

$$M:0 = a+c+1$$
 ......(14)  
 $L:0 = -a+2b-c-2$  ......(15)  
 $T:0 = -2a-c-1$  ......(16)  
From equation (iii),  $c = -2a-1$  ............(17),

Combining equation (14) and (16), => a-2a-1+1=0

$$=$$
è  $a = 0, c = -1$ 

Solving equation (15) for the values of a=0 and c=-1 yields, b=  $\frac{1}{2}$ 

Substituting the values of a, b and c in Equation (7), we obtain:

$$\Pi_1 = \frac{A^{1/2}\gamma}{\mu} \dots (18)$$

Similarly, analyzing  $\pi_2$ ,  $\pi_3$ ,  $\pi_4$ ,  $\pi_5$  and  $\pi_6$  terms respectively

gives:

$$\pi_2 = \frac{R\mu^2}{P}$$
,  $\pi_3 = \frac{PAC}{\mu^2}$ ,  $\pi_4 = \frac{P\theta}{\mu}$ ,  $\pi_5 = \frac{V}{A^{3/2}}$ ,  $\pi_6 = PS$ 

Substituting the specific expressions for the dimensionless

terms;  $\pi_1$ ,  $\pi_2$ ,  $\pi_3$ ,  $\pi_4$ ,  $\pi_5$  and  $\pi_6$  into equation (6), yields:

$$f\left(\frac{A^{1/2}Y}{\mu}, \frac{\mu^2 R}{P}, \frac{PAC}{\mu^2}, \frac{P\theta}{\mu}, \frac{V}{A^{3/2}}, PS\right) = 0 \dots (19)$$

Since equation (19) does not give the exact relationship between the parameters being investigated; there is need to generate experimental data. Following Buckingham's  $\pi$ -method, any of the dimensionless terms of equation (19) can be written as a function of the others hence, it transforms to:

$$\frac{\mu^2 R}{P} = K \left( \frac{A^{1/2} Y}{\mu} \right)^a \left( \frac{PAC}{\mu^2} \right)^b \left( \frac{P\theta}{\mu} \right)^c \left( \frac{V}{A^{3/2}} \right)^d (PS)^e \dots (20)$$

The exponents in equation (20) can be obtained by regression analysis using experimental data.

For easy determination of the exponents, the above equation can be transformed as;

$$\operatorname{Ln} \frac{\mu^2 R}{P} = \operatorname{Ln} K + \operatorname{aLn} \frac{A^1 I^2 Y}{\mu} + \operatorname{bLn} \frac{PAC}{\mu^2} + \operatorname{cLn} \frac{P\theta}{\mu} + \operatorname{dLn} \frac{V}{A^3 I^2} = \operatorname{Ln} PS....$$
 (21)

## **ASSUMPTIONS**

Let M = 
$$Ln\frac{\mu^2R}{P}$$
 ,  $X_1=\left(\frac{A^{1/2}Y}{\mu}\right)$  ,  $X_2=Ln\frac{PAC}{\mu^2}$  ,  $X_3=Ln\frac{P\theta}{\mu}$  ,  $X_4=Ln\frac{P\theta}{\mu}$ 

$$Ln\frac{V}{A^3I^2}$$
, and X<sub>5</sub> = PS

Hence, equation (21) becomes,

$$M = Ln K + aX_1 + bX_2 + cX_3 + dX_4 + eX_5.....$$
 (22)

From the experimental data obtained (Data too large to reproduce), values of the constants a, b c, d and e were evaluated by Regression using SPSS (Table 2).

Table 2: LMT Model Coefficients

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
Intercept	2.3450	0.9835	2.3843	0.0206	0.3740	4.3159
0.0884	-0.8073	0.0769	10.504	0.0000	0.9613	0.6533
21.0151	-0.1178	0.0624	1.8882	0.0643	0.2429	0.0072
19.0380	-0.0459	0.0480	0.9567	0.3429	0.1420	0.0502
-1.8157	-1.5772	0.3955	3.9877	0.0002	2.3699	0.7846
7.0733	0.0350	0.0301	1.1653	0.2489	0.0252	0.0953

From Table 2 above, LnK = 2.345, è K = 10.4332

But 
$$\left(\frac{\mu^2 R}{P}\right) = K\left(\frac{A^{1/2} Y}{\mu}\right)^a \left(\frac{PAC}{\mu^2}\right)^b \left(\frac{P\theta}{\mu}\right)^c \left(\frac{V}{A^{3/2}}\right)^d (PS)^e$$
,

Hence, substituting the values of K, a, b, c, d and e in equation (20) yields,

$$\frac{\mu^2 R}{P} = K(\frac{A^{\frac{1}{2}}Y}{\mu})^{-0.807} (\frac{PAC}{\mu^2})^{-0.118} (\frac{P\theta}{\mu})^{-0.0459} (\frac{V}{A^{\frac{3}{2}}})^{-1.577} (PS)^{0.035} \dots (23)$$

$$R = \frac{P^{0.871} S^{0.035} A^{1.844}}{C^{0.1178} \mu^{0.911} \theta^{0.0459} \gamma^{1.5772} \gamma^{0.8073}} \dots (24)$$

Equation (24) can be transformed as follows: By dividing both sides of equation (24) by  $\theta$  and rearranging,

$$\frac{V^{1.5772}}{\theta} = \frac{10.4332 \, x \, P^{0.871} \, S^{0.035} A^{1.844}}{R \, Y^{0.8073} \, \mu^{0.911} \, C^{0.1178}} \, x \, \frac{1}{\theta^{1.0459}} \dots \tag{25}$$

A plot of  $\frac{V^{1.5772}}{\theta}$  and  $\frac{1}{\theta^{1.0459}}$  gives a straight line with slope,  $b_2 = 71.653$ 

From the graph (Figure 2), slope,  $b_2 = 71.653$ 

Hence, equation (25) becomes,

Equation (26) is the desired Cake Yield Equation. Substituting the values of b<sub>2</sub> and K in the model yields,

$$R_{LMT} = 0.1456 \times \frac{P^{0.871} S^{0.035} A^{1.844}}{Y^{0.8073} \mu^{0.911} C^{0.1178}} \dots (27)$$

#### 3. Results and Discussion

# 3.1 Evaluation of the Filter Cake Yields, Specific Resistance and Compressibility Coefficient.

The formulation of the model involved nine (9) parameters listed in equation (5) above. The first six (6) were experimentally determined while the last three were evaluated. Cake yield was evaluated using data generated from the pressure filtration of the Brewery Sludge Samples from Nigerian Brewery, 9th Mile, Enugu, Nigeria (Data too large to reproduce here). For the purpose of the parameters mentioned above, a detailed analyses of the raw data obtained were made. The slopes of both t/V against V and t/V against t respectively for specific resistance and experimental solids yields were obtained using graphical methods. Moreover, Table 3 values were used for the evaluation of slope, b2 in equation (26).

Table 3: Data for the plot of  $\frac{V^{1.5772}}{\theta}$  and  $\frac{1}{\theta^{1.0459}}$ 

<b>θ</b> (s)	V(Cm³)	<b>V</b> 1.5772	$V^{1.5772}/\Theta$	1/ <b>0</b>
		(Cm <sup>4.73</sup> )	(Cm <sup>4.73</sup> /S)	(1/S)
10	18	95.4588	9.5459	0.0900
20	26	170.4888	8.5244	0.0436
30	34	260.2846	8.6762	0.0285
40	36	284.8396	7.1210	0.0211
50	38	310.1949	6.2039	0.0167
60	40	336.3326	5.6055	0.0138
70	42	363.2359	5.1891	0.0118
80	44	390.8891	4.8861	0.0102
90	46	419.2776	4.6586	0.0090
100	48	448.3877	4.4839	0.0081
110	50	478.2065	4.3473	0.0073

120	52	508.7219	4.2393	0.0067
130	54	539.9225	4.1533	0.0062
140	55	555.7763	3.9698	0.0057

# 3.2 Variation of Specific Resistance with operating Pressures

According to the relationship derived from Darcy's law which relates pressure drop to specific resistance to filtration (equation 3), an increase in pressure drop should result in an increase in specific resistance. This is the case if the filter cake is not highly compressible such that the specific cake resistance increases with pressure drop [21]. It is also beneficial to gradually increase the pressure until a constant pressure is reached. This is because the solids are non-homogeneous and a high initial pressure drop can result in particles plugging the interstices of the cloth. With the assumption that the cake was not highly compressible, the applied pressure was thus set to a maximum of 6.5bars for all runs.

The graph in figure 4 shows that pressures increases as the operating pressure increases, which is in agreement with both Carman and Ademiluyi findings earlier cited above.

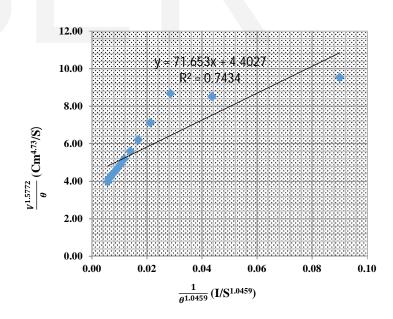


Fig. 3: A plot of  $\frac{v^{1.5772}}{\theta}$  and  $\frac{1}{\theta^{1.0459}}$  for the determination of slope, b<sub>2</sub>

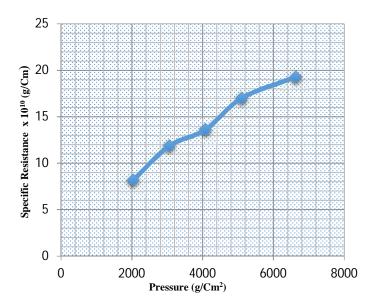


Fig. 4: Effects of Operating Pressures on Specific Resistance.

Mathematically from the model,

$$R \propto P^{0.8713}$$

Also, from Figure 4, increasing the operating pressure from 3000g/cm<sup>2</sup> to 6628g/cm<sup>2</sup> increased the cake yield by 1.63 times.

Furthermore, the deterioration of filtrate quality as the pressures were increased cannot be ignored as was the case with previous Researchers. However, physically, it is quite easy to explain. As the operating pressures were increased, sludge flocs were ruptured accounting for the poor filtrate quality.

# 3.3. Variation of specific resistance with Conditioner dosages

From Figures 5, specific resistance decreases with increased conditioner dosage until an optimum dosage is reached, all other conditions being equal. For instance, increasing Ferric Chloride dosage from 13% to 21.2% decreased specific resistance from 19.2 g/Cm to 18.15g/Cm The optimum dosages from the graph to attain acceptable filtrate quality was 22.03% at an operating pressure of 6628.18g/Cm<sup>2</sup>. Considering the differences in the concentrations of the five sludge samples tested, it is significant that they all responded similarly. Also, the

decrease in the specific resistance may be attributable to the reduction in sludge compressibility due to the increased conditioner doses.

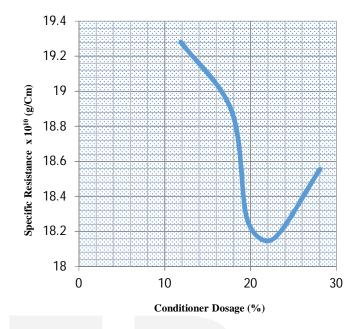


Fig. 5: Effects of Conditioner Dosages on Specific Resistance

It is important to note that overdosing the unconditioned sludge beyond the optimum requirement mares the solids yield while the possibility of increasing the specific resistance cannot be ruled out. This is due to de-flocculation as a result of excessive surface coverage and charge reversal. Overall, the most important benefit of polymer conditioning of brewery sludge is the improvement in sludge dewaterability. This benefit is based on the proper use of polymers and their integrated effects on sludge characteristics.

# 3.4 Variation of Specific Resistance with Initial Solids Content

Many Researchers including [22] and [23] have indicated this filterability dependence on initial solids content especially when considering the effects on specific resistance on filtration, but the effect was never reported to be as great as observed in this study.

The variation of specific resistance for different values of initial solids content at different operating pressures is shown in Figure 6. It is important to note here that the effect of initial solids moisture on performance is much more pronounced in Filter Presses than in Vacuum

filtration, [24]. Specific resistance decreases with increased initial solids content at higher pressures. From the developed model, specific resistance is mathematically related to operating initial solids content, C as shown below:

$$R = K/C^{0.1178}$$

Where K is proportionality constant.

Similarly, results show that an increase in the concentration of solids in the feed results in an increase in dry cake production and that an increase in the specific cake resistance can result in a decrease in the dry cake production.

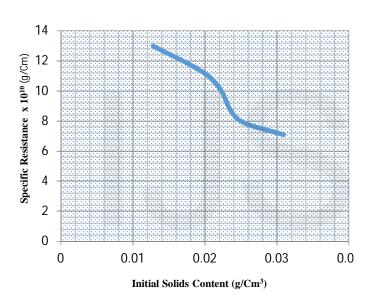


Fig.6: Variation of Initial Solids Content with Specific Resistance

The relationship between the specific resistance to filtration and the solids concentration is very nearly accurate to the relationship described above. Deviation from linearity can be due to the effect of the filter medium resistance, which is neglected in the derivation of the relationship.

The benefit of a higher feed solids concentration can be seen in the resultant reduction in specific cake resistance.

## 3.5 Variation of Specific Resistance with Filter Cake

Yield

The developed model predicts that more solids are captured on the filter as specific resistance decreases. The effect of specific resistance on yield is shown on Figure 7. However, the reason for this is that more cake are deposited when there is less restriction to filtration, taking into account other conditions such filtration pressures, time and conditioner dosages. This can be mathematically represented as follows;

$$R = k/Y^{0.8073}$$

Where k is proportionality constant given as

**0.1456** 
$$x = \frac{P^{0.8713} S^{0.035} A^{1.844}}{\mu^{0.9112} C^{0.119}}$$

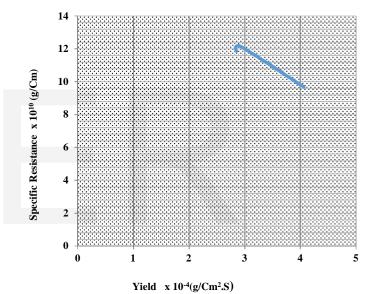


Fig. 7: Variation of Specific Resistance with Filter Cake Yield

In other words, cake yield is inversely proportional to specific resistance. This is in agreement with the findings of [25],[26], [24] and [14.The implication of the above is that all other conditions making up the proportionality constant must be in place for the relation to be valid.

3.6 Variation of Specific Resistance with Compressibility From the developed equation, specific resistance was seen to increase correspondingly with increased compressibility for 0.02645g/Cm<sup>3</sup> tested sludge sample. Figure 8 shows specific resistance increasing with compressibility but falls steeply when compressibility value increased above 0.7981Cm.S<sup>2</sup>/g. The initial rise in specific resistance value

with compressibility may be attributable to other operating conditions such as pressure and chemical dosage. In summary, the graph agrees with the model theoretical prediction given as;

 $R = kS^{0.0350}$ 

Where: k is proportionality constant.

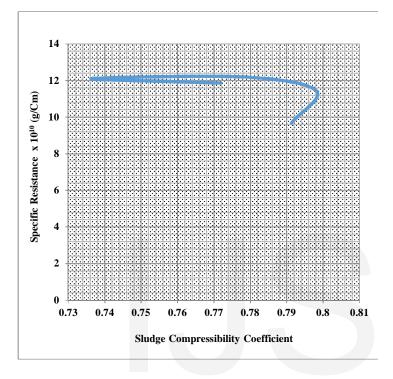


Fig. 8: Variation of Specific Resistance with Sludge

Compressibility

#### 4. Conclusion

The LMT Model equation developed (Equation 27) shows that the specific resistance to filtration for a filter press is directly proportional to the filter area of the pressure vessel, applied pressure and compressibility coefficient of the sludge while being inversely proportional to filter cake yield, viscosity of filtrate, and initial solids content of the sludge.

This is in agreement with scientific reasoning and experimental observation. Equations 26 and 27 enable performance of a pressure filter (Filter Press) to be predicted from a simple laboratory determination of both cake yields and specific resistance simultaneously. Curves derived from equation 27 show how specific resistance to

filtration depends on a number of parameters, for instance, figures 4,5,6,7 and 8 illustrate the dependence of specific resistance on operating pressures, conditioner dosages, initial solids contents, cake yield and sludge compressibility.

Moreover, experimental verification of the equation and the derived curves has been described and it may be concluded that for practical purposes, the predicted performance agrees with measured values. What makes the model a novelty is the incorporation of the compressibility and filter yield attributes of the sludge.

Moreover,[17] had stated that the only way through which a filtration can validly predict a filtration process is when the equation's plot of t/V and V gives a straight line, the developed model agreed with the above assertion as the plot of  $\frac{V^{1.5772}}{\theta}$  and  $\frac{1}{\theta^{1.0459}}$  gave s straight line.

## 5. Acknowledgements

The Authors wish to acknowledge the Lord God Almighty for the grace and for making this work a huge success. The moral support and encouragement towards realizing the objectives of the study from Chief and Mrs. J. O. Igboke, are acknowledged. The authors would like to thank Professor J.C Agunwamba , Dr. C. C Nnaji of the Department of Civil Engineering, University of Nigeria, Nsukka for the their valuable comments during this study and the preparation of this paper..

A big thanks to all the staff of Sanitary Engineering Laboratory for assisting me in the course of carrying out some of the experiments.

Finally, I wish to acknowledge the support and encouragement I got from my Departmental Staff in the Federal Ministry of Environment, Abuja, They include: Engr. E.J. Ekanem, (Fmr. Head, Flood Monitoring and Control), Mr. U.T. Egwuatu and Engineers: M. Kopada, Subuloye, Iheanacho amongst others.

To God be Glory.

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